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Total No. of Questions : 09

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B.Tech. (Automation & Robotics)/ (CSE)/ (Electrical & Electronics Engineering)/ (EE)/ (ECE)/ (Electronics & EE) (Sem–2)

MATHEMATICS-II

Subject Code : BTAM202-18

M.Code : 91958

Date of Examination : 13-06-2023

Time : 3 Hrs.

Max. Marks : 60

INSTRUCTIONS TO CANDIDATES :

1. SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.
2. SECTION - B & C. have FOUR questions each.
3. Attempt any FIVE questions from SECTION B & C carrying EIGHT marks each.
4. Select atleast TWO questions from SECTION - B & C.

SECTION-A

1. Solve :

a) For the differential equation $(e^{2y} + 1) \cos x\, dx + 2e^{2y} \sin x\, dy = 0$, check whether the equation is exact or not.

b) Find the general solution of the first order linear differential equation $y' + y = \sin x$.

c) Find the general solution of the Clairaut's equation $y = xy' - (1/y')$.

d) Find the general solution of the differential equation $y'' + 8y' - 9y = 0$.

e) Find the solutions of the homogeneous partial differential equation:

$$\left[2D^2 + 5DD' + 3(D')^2 + D + D'\right]z = 0, \text{ where } z = f(x, y).$$

f) Find an interval which contains the root of the equation: $x^{ex} - 1 = 0$.

g) Construct the forward difference table for the data

x	-4	-2	0	2	4	6
$f(x)$	-67	-9	1	11	69	223

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h) What is Trapezoidal rule. Give its formula.

i) Write down Laplace equation in two variables.

j) State Milne's Predictor-Corrector method.

SECTION-B

2. a) Solve the initial value problem $(\cos x + y\sin x)dx - (\cos x)dy$, $y(\pi) = 0$.

   b) Find the solution of the Bernoulli equation $xy' = (y^2 - 1)/y$.

3. Find the general solution of the differential equation $y'' + 4y = \cos x$, using the method of variation of parameters.

4. Find the general solutions of the partial differential equation:

   $[6D^2 + 5DD' - 6(D')^2]z = 132\log(x + 3y)$.

5. Find the complete integral of the partial differential equation $p^2 - 3q^2 = 5$.

SECTION-C

6. Perform three iterations of the Newton-Raphson method to find a root of the equation $xe^x - 1 = 0$, which is close to 0.5.

7. Evaluate $\int_1^2 \frac{x^2}{1+x^3}dx$ using the Simpson's l/3rd rule with four sub-intervals. Compare with the exact solution.

8. Solve the initial value problem $y' = x(y - x)$, $y(2) = 3$ in the interval [2,2.4] using the classical Runge-Kutta fourth order method with the step size $h = 0.2$.

9. In the initial value problem $y' = xy + x^2y^2 + 1$, $y(1) = 2$, $h = 0.1$, $x \in [1,1.3]$, find the approximate values of $y(x)$ at the given point using the Euler method.

NOTE : Disclosure of Identity by writing Mobile No. or Marking of passing request on any paper of Answer Sheet will lead to UMC against the Student.

Mathematics - II

Subject Code :- BTAM202-18

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Section - A

1.

a) $(e^{2y}+1)\cos x\, dx + 2e^{2y}\sin x\, dy = 0$ — (1)

comparing (1) with $M dx + N dy = 0$

$M = (e^{2y}+1)\cos x$; $N = 2e^{2y}\sin x$

$\frac{\partial M}{\partial y} = 2e^{2y}\cos x$; $\frac{\partial N}{\partial x} = 2e^{2y}\cos x$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$\therefore$ equation (1) is exact.

b) $y' + y = \sin x$

$\frac{dy}{dx} + y = \sin x$ — (1)

comparing (1) with $\frac{dy}{dx} + Py = Q$

$P = 1$; $Q = \sin x$

Integrating factor :- $e^{\int P\,dx} = e^{\int 1\,dx} = e^{x}$

solution of (1) is given by,

$$y \times I.F = \int Q \times I.F \, dx + c$$

$$y e^x = \int e^x \sin x \, dx + C$$

$$y e^x = \frac{e^x}{2} (\sin x - \cos x) + c.$$

(c) $y = x y' - \frac{1}{y'}$

$$y' = \frac{dy}{dx} = p$$

$y = px - \frac{1}{p}$, which is clairaut's equation,

Its solution is given by put $p = c$

$$\boxed{y = cx - \frac{1}{c}} \text{ - Ans.}$$

(d) $y'' + 8y' - 9y = 0$

$$[D^2 + 8D - 9] y = 0$$

$$D^2 + 8D - 9 = 0$$

$$D^2 - 9D + D - 9 = 0$$

$$D(D-9) + 1(D-9) = 0$$

$(D-9)(D+1)=0$

$\Rightarrow D = 9, -1$

C.F:- $y = C_1 e^{9x} + C_2 e^{-x}$.

(2) $[2D^2 + 5DD' + 3(D')^2 + D + D']z = 0$

$2D^2 + 5DD' + 3(D')^2 + D + D' = 0$

$2D^2 + 2DD' + 3DD' + 3(D')^2 + D + D' = 0$

$2D(D+D') + 3D'[D+D'] + (D+D') = 0$

$(D+D')(2D+3D'+1) = 0$.

C.F:- $\phi_1(x-y) + e^{-\frac{1}{2}x}\phi_2(3x-2y)$.

$\because$ If

$f(D,D') = (\alpha D_x + \beta D' + \gamma)$

C.F:- $e^{-\frac{\gamma}{\alpha}x}\phi(\beta x - \alpha y)$

(3) $x^{e^x} - 1 = 0$.

Soln:- $f(x) = x^{e^x} - 1$

$f(0) = -1$

$f(1) = 0$

so root of given equation lies in interval $\boxed{(0,1)}$ → Ans

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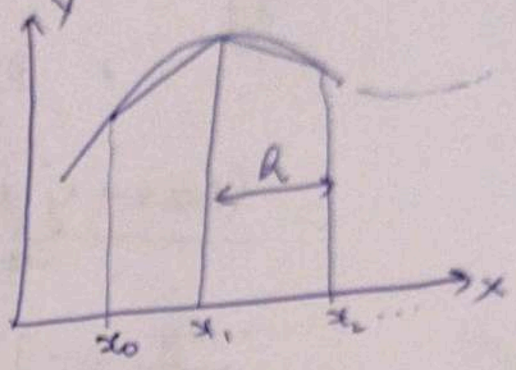
x	$f(x) = y$	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
-4	-67					
		58	-48	48		
-2	-9	10			$+0$	
0	1		0			0
		10		$+48$		
2	11		48		0	
		58		$+48$		
			$+96$			
4	69					
		154				
6	223					

(h) Trapezoidal Rule :- To approximate the curve $y = f(x)$ between $x = x_0$, $x = x_n$ by 'n' straight lines and calculate area of each of 'n' trapezium

The value of $\int_{x_0}^{x_n} f(x)\,dx$



is given by,

$$\int_{x_0}^{x_n} f(x)\,dx = \frac{h}{2}\left[y_0 + 2(y_1 + y_2 + \dots) + y_n\right]$$

(i) Laplace equation in two variables is given by,

$$u_{xx} + u_{yy} = 0.$$

(j) Milne's Predictor - Corrector method is given by,

$$y^{p}_{n+1} = y_{n-3} + \frac{4h}{3}\left[2 f_{n-2} - f_{n-1} + 2 f_n\right]$$

$$y^{c}_{n+1} = y_{n-1} + \frac{h}{3}\left[f_{n-1} + 4 f_n + f^{p}_{n+1}\right] ; \quad p = \frac{x - x_{n-1}}{h}$$

Section - B

(2) (a) $(\cos x + y \sin x)\,dx - \cos x\,dy = 0;\quad y(\pi)$

$\cancel{M\,dx + N\,dy = 0}$

$\cancel{M = \cos x + y \sin x \;;\; N = -\cos x}$

$\cancel{\frac{\partial M}{\partial y} = \cos x \;;\; \frac{\partial N}{\partial x} = \sin x}$

$\cancel{\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}}$, ~~given eqn is not exact.~~

$$\frac{(\cos x + y \sin x)}{\cos x} = \frac{dy}{dx}$$

$$\frac{dy}{dx} = 1 + y \tan x \Rightarrow \frac{dy}{dx} - y \tan x = 1 \quad \text{—(1)}$$

comparing (1) with $\frac{dy}{dx} + Py = Q$, $P = -\tan x$, $Q = 1$

I.F :- $e^{\int P\,dx} = e^{-\int \frac{\sin x}{\cos x}dx} = e^{\int \frac{1}{t}dt} = e^{\log t} = e^{\dots} = \cos x$

Solution of (1) is given by

$y \times I.F = \int Q \times I.F\,dx + C$

$y \cos x = \int \cos x\,dx + C \Rightarrow \boxed{y \cos x = \sin x + C}$

$y = \tan x + C \sec x$

now put $y(\pi) = 0$

$0 = 0 + C(-1) \Rightarrow \boxed{C = 1}$

$\therefore$ complete soln is :- $\boxed{y = \tan x + \sec x}$

(b) $x y' = \frac{y^2 - 1}{y}$

soln:- $y' x = y - \frac{1}{y}$

or $y' = \frac{y}{x} - \frac{1}{xy}$

$y' - \frac{y}{x} = -\frac{1}{x} \cdot y^{-1}$ —(1) $\Rightarrow y y' - \frac{1}{x} y^2 = -\frac{1}{x}$

Put $y^2 = t$

$2yy' = t'$

$\boxed{\frac{1}{2} t' - \frac{1}{x} t = -\frac{1}{x}}$

or $\boxed{t' - \frac{2}{x} t = -\frac{1}{x}}$ —(3)

what is Leibniz Linear D.E

$y' + P(x) y = Q(x) \cdot y^n$ —(2)

comparing (1) & (2)

$P(x) = -\frac{1}{x}$; $Q(x) = -\frac{1}{x}$, $n = -1$

I.F :- $e^{\int (1-n) P(x) dx} = e^{2\int -\frac{1}{x} dx} = e^{-2\log x} = e^{\log x^{-2}} = \boxed{\frac{1}{x^2}}$

soln of (3) is given by

~~$y^{1-n} \times I.F = \int (1-n) Q(x) \times I.F\,dx + c$~~

$y \times I.F = \int Q(x) \times I.F\,dx + C$

$y \times \frac{1}{x^2} = \int -\frac{1}{x} \times \frac{1}{x^2} dx + C$

$\frac{y}{x^2} = -\int x^{-3} dx + C$

$\frac{y}{x^2} = + \frac{x^{-2}}{2} + c$

$$\boxed{y = \frac{1}{2} + C x^2}$$

3) $y'' + 4y = \cos x$

Symbolic form :- $[D^2+4]y = \cos x$

A.E :- $D^2 + 4 = 0$

$D^2 = -4$

$D = \pm 2i$

$\therefore C.F = C_1 \cos 2x + C_2 \sin 2x$

Let $y_1 = \cos 2x$; $y_2 = \sin 2x$

$y_1' = -2\sin 2x$; $y_2' = 2\cos 2x$

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$

$$= \begin{vmatrix} \cos 2x & \sin 2x \\ -2\sin 2x & 2\cos 2x \end{vmatrix}$$

$$= 2(\cos^2 2x + \sin^2 2x) = 2$$

$\therefore W = 2$

$$\therefore I = -y_1 \int \frac{y_2}{W} X \, dx + y_2 \int \frac{y_1}{W} X \, dx$$

$$= -\cos 2x \int \frac{\sin 2x}{2} \cos x \, dx + \sin 2x \int \frac{\cos 2x}{2} \cos x \, dx$$

$$= -\frac{\cos 2x}{4} \int (\sin 3x + \sin x) \, dx + \frac{\sin 2x}{4} \int (\cos 3x + \cos x) \, dx$$

$$= -\frac{\cos 2x}{4}\left[-\frac{\cos 3x}{3} - \cos x\right] + \frac{\sin 2x}{4}\left[\frac{\sin 3x}{3} + \sin x\right].$$

$$= \frac{\cos 2x \cos 3x}{12} + \frac{\cos 2x \cos x}{4} + \frac{\sin 2x \sin 3x}{12} + \frac{\sin 2x \sin x}{4}$$

$$= \frac{1}{12}\left[\cos 3x \cos 2x + \sin 3x \sin 2x\right] + \frac{1}{4}\left[\cos 2x \cos x + \sin 2x \sin x\right].$$

$$= \frac{1}{12}\left[\cos(3x - 2x)\right] + \frac{1}{4}\cos(2x - x)$$

$$= \frac{\cos x}{12} + \frac{\cos x}{4} = \frac{\cos x + 3\cos x}{12} = \frac{4}{12}\cos x$$

$$= \boxed{\frac{\cos x}{3}} \rightarrow \text{Ans.}$$

(4) $\left[6D^2 + 5DD' - 6(D')^2\right]z = 132 \log(x + 3y)$

Put $D = m$, $D' = 1$

<u>A.E</u>:- $6m^2 + 5m - 6 = 0$

$$m = \frac{-5 \pm \sqrt{25 + 144}}{12}$$

$$= \frac{-5 \pm 13}{12}$$

$$= \frac{-5 - 13}{12}, \frac{-5 + 13}{12}$$

$$= -\frac{18}{12}, \frac{8}{12}$$

$$m = -\frac{3}{2}, \frac{2}{3}$$

$6D^2 + 9DD' - 4DD' - 6D'^2$

$3D[2D + 3D'] - 2D'[2D + 3D']$

$[2D + 3D'][3D - 2D']$

<u>C.f</u>:-

$\phi_1(2y - 3x) + \phi_2(3y + 2x)$

Or

$\therefore$ <u>C.f</u> :- $\phi_1\left(y - \frac{3}{2}x\right) + \phi_2\left(y + \frac{2}{3}x\right)$

$\underline{P.I}$:- $\dfrac{1}{6D^2+5DD'-6(D')^2}\, 132\log(x+3y)$

$= 132\dfrac{1}{(2D+3D')(3D-2D')}\log(x+3y)$

Put $D=1,\ D'=3$

$= 132\dfrac{1}{(2+9)(3-2)}\log(x+3y)$

$= \dfrac{132}{11}\log(x+3y) = 12\log(x+3y)$

$\therefore$ complete solution is given by,

$y = C.F + P.I$

$$\boxed{y = \phi_1(2y-3x) + \phi_2(3y+2x) + 12\log(x+3y)} \rightarrow \underline{Ans}.$$

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(5) $p^2 - 3q^2 = 5$

Sol:- S.f $\therefore$ $D^2 - 3D'^2 = 0$

$\underline{A.E}$:- $D^2 - 3D'^2 = 0$

Put $D=m,\ D'=1$

$m^2 - 3 = 0$

$m^2 = 3$

$m = \pm\sqrt{3}$

$\underline{C.F}$:- $\phi_1(y+\sqrt{3}x) + \phi_2(y-\sqrt{3}x)$

(5) $\cancel{p^2 - 3q^2 = 5}$

$$5 = \frac{1}{D^2 - 3D'^2} 5 e^{0x+0y}$$

P.I :- $y = \frac{1}{D^2 - 3D'^2}$

$= 5 \frac{1}{D^2 - 3D'^2} e^{0x+0y}$, Put $D = 0, D' = 0$ which is case of failure,

$= 5x \frac{1}{2D} e^{0x+y0}$, Again case of failure

$= \frac{5x^2}{2 \cdot 2!} = \frac{5}{4} x^2.$

$\therefore$ complete soln is given by,

$$y = C.F + P.I$$

$$\boxed{y = \phi_1(y + \sqrt{3}x) + \phi_2(y - \sqrt{3}x) + \frac{5}{4}x^2}$$

Section - C

6) $\boxed{f(x) = xe^x - 1}$ — (1)

$f'(x) = xe^x + e^x$

$\boxed{f'(x) = e^x(x+1)}$ — (2)

$f(0) = -1 < 0$
$f(1) = 0 > 0$
$\therefore$ root lies between $(-1, 0)$

let $x_0 = 0.5$

using Newton Raphson Method,

$$x_{n+1} = x_{n-1} - \frac{f(x_{n-1})}{f'(x_{n-1})} \quad — (3)$$

$$\therefore x_1 = x_0 - \frac{f(x_0)}{f'(x_0)},$$

$$= 0.5 - \frac{(-0.1756)}{2.473}$$

$f(0.5) = 0.5e^{0.5} - 1$
$= \cancel{7.2436}$ -0.1756

$f'(0.5) = e^{0.5}(0.5+1)$
$= 2.473$

$= 0.5 + 0.07100$
$= 0.571068$
$= 0.5710068$

Now $f(x_1) = 0.1070958$ [using (1) & (2)]
$f'(x_1) = 2.780758$

$$\therefore x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$ [using (3)]

$$= 0.5710068 - \frac{(0.1070958)}{2.780758}$$

$x_2 = 0.53249$

Now $f(x_2) = -0.09308$ [using (1) & (2)
$= 2.610088$
$f'(x_2) = 2.610088$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} \quad [\text{from } 3]$$

$$= 0.53249 - \frac{(-0.09308)}{2.61088}$$

$$= 0.56815$$

$$f(x_3) = 0.00275$$
$$f'(x_3) = 2.7678 \quad [\text{from } 1 \text{ \& } 2]$$

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)} \quad [\text{from } 3]$$

$$= 0.56815 - \frac{0.00275}{2.7678}$$

$$x_4 = 0.56716$$

$$f(x_4) = 0.0000462$$
$$f'(x_4) = 2.7633$$

$$x_5 = x_4 - \frac{f(x_4)}{f'(x_4)}$$

$$= 0.56716 - \frac{0.0000462}{2.7633}$$

$$x_5 = 0.56714$$

As ~~two~~ x_4 & x_5 are same upto 4 decimal places

$\therefore$ root of given equation is

$$\boxed{x = 0.5671} \rightarrow \text{Ans.}$$

(7) $\int_1^2 \frac{x^2}{1+x^3}\,dx$

$y = f(x) = \frac{x^2}{1+x^3}$

Here $n = 4$; $x_0 = 1$; $x_n = 2$

$\therefore h = \frac{x_n - x_0}{n} = \frac{2-1}{4} = \frac{1}{4} = 0.25$

x	1 (x_0)	1.25 (x_1)	1.5 (x_2)	1.75 (x_3)	2 (x_4)
$f(x) = y$	0.5 (y_0)	0.5291 (y_1)	0.5143 (y_2)	0.4816 (y_3)	0.4444 (y_4)

By Simpson $\frac{1}{3}$ Rule

$$\int_1^2 \frac{x^2}{1+x^3}\,dx = \frac{h}{3}\left[y_0 + y_4 + 4(y_1 + y_3) + 2y_2\right]$$

$$= \frac{0.25}{3}\left[0.5 + 0.4444 + 4(0.5291 + 0.4816) + 2 \times 0.5143\right]$$

$$\boxed{\int_1^2 \frac{x^2}{1+x^3} = 0.5013} \quad - (1)$$

Now for exact solution,

$$\frac{1}{3}\int_1^2 \frac{3x^2}{1+x^3}\,dx$$

Put $1+x^3 = t$ | when $x=1$, $t=2$

$3x^2\,dx = dt$ | when $x=2$, $t=9$

$$\frac{1}{3}\int_2^9 \frac{1}{t}\,dt$$

$$\frac{1}{3}\left|\log t\right|_2^9$$

$$= \frac{1}{3}\left[\log 9 - \log 2\right]$$

$$= \boxed{0.2177} \quad (2)$$

comparing (1) & (2)

$\text{error} = 0.5013 - 0.2177$

$= 0.2836$

(8) $y' = x(y-x)$

$y(2) = 3$ $\quad [2, 2.4]$

$h = 0.2$

$f(x) = x(y-x)$ $\quad x_0 = 2;\ y_0 = 3,\ h = 0.2$

Step 1:-

$k_1 = h\, f(x_0, y_0)$

$= 0.2\, f(2, 3)$

$= 0.2[2] = 0.4$

$k_2 = h\, f\left(x_0 + \frac{h}{2},\ y_0 + \frac{k_1}{2}\right)$

$= 0.2\, f(2.1,\ 3.2)$

$= 0.2 \times 2.1 \times (3.2 - 2.1)$

$= 0.2 \times 2.1 \times 1.1$

$= 0.462$

$k_3 = h\, f\left(x_0 + \frac{h}{2},\ y_0 + \frac{k_2}{2}\right)$

$= 0.2\, f(2.1,\ 3.231)$

$= 0.47502$

$k_4 = h\, f(x_0 + h,\ y_0 + k_3)$

$= 0.2\, f(2.2,\ 3.47502)$

$= 0.56101$

$y_1 = y_0 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

$= 3 + \frac{1}{6}[0.4 + 0.462 + 0.47502 + 0.56101] = 3.4725$

$\therefore y(2.2) = 3.47255$

Step 2:- Let $x_1 = 2.2$; $y_1 = 3.4725$, $h = 0.2$

$k_1 = h f(x_1, y_1)$

$= 0.2 f(2.2, 3.4725)$

$= 0.5599$

$k_2 = h f\left(x_1 + \frac{h}{2}, y_1 + \frac{k_1}{2}\right)$

$= 0.2 f(2.3, 3.752445)$

$= 0.66813$

$k_3 = h f\left(x_1 + \frac{h}{2}, y_1 + \frac{k_2}{2}\right)$

$= 0.2 f(2.3, 3.80656)$

$= 0.69302$

$k_4 = h f(x_1 + h, y_1 + k_3)$

$= 0.2 f(2.4, 4.16552)$

$= 0.84745$

$\therefore y_2 = y_1 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

$= 3.4725 + \frac{1}{6}[0.5599 + 2(0.66813) + 2(0.69302) + 0.84745]$

$= 4.1608$

$\therefore \boxed{y(2.4) = 4.1608}$ → Ans.

$y' = xy + x^2y^2 + 1 = f(x, y)$

$y(1) = 2\ ;\ h = 0.1$ $\qquad x \in [1, 1.3]$

Here $x_0 = 1$

$y_0 = 2$

Step 1:- $y_1 = y_0 + h\, f(x_0, y_0)$

$= 2 + 0.1\ f(1, 2)$

$= 2 + 0.1\left[1 \times 2 + 1^2 \cdot 2^2 + 1\right]$

$= 2 + 0.1\left[2 + 4 + 1\right]$

$= 2 + 0.7 = 2.7$

$\therefore y_1(1.1) = 2.7$

Step 2:- $x_1 = 1.1\ ;\ y_1 = 2.7$

$y_2 = y_1 + h\ f(x_1, y_1)$

$= 2.7 + (0.1)\ f(1.1, 2.7)$

$= 2.7 * (0.1)\left[(1.1 \times 2.7) + (1.1)^2 \times (2.7)^2 + 1\right]$

$= 2.7 + 7.808$

$= \cancel{10.508}\ 3.979$

$\therefore y_2(1.2) = \cancel{10.508}\ 3.979$

Step 3:- $x_2 = 1.2\ ;\ y_2 = \cancel{10.508}\ 3.979$

$y_3 = y_2 + h\ f(x_2, y_2)$

$= 10.\cancel{508} + 0.1 f(1.2; 10.508)$

$= 10.\cancel{508} + 0.1 \left[(1.2 \times 10.508) + (1.2)^2 \times (10.508)^2 + 1 \right]$

$= 3.979 + 0.1 f(1.2, 3.979)$

$= 3.979 + 0.1 \left[(1.2 \times 3.979) + (1.2)^2 \times (3.979)^2 + \right.$

$= 3.979 + 2.8574$

$= 6.8364$

$\therefore \boxed{y(1.3) = 6.8364} \rightarrow$ Ans.